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# Low-scale seesaw mechanisms for light neutrinos

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## Abstract

Alternatives to the seesaw mechanism are explored in supersymmetric models with three right-handed or sterile neutrinos. Tree-level Yukawa couplings can be drastically suppressed in a natural way to give sub-eV Dirac neutrino masses. If, in addition, a $B-L$ gauge symmetry broken at a large scale $M_G$ is introduced, a wider range of possibilities opens up. The value of the right-handed neutrino mass $M_R$ can be easily disentangled from that of $M_G$. Dirac and Majorana neutrino masses at the eV scale can be generated radiatively through the exchange of sneutrinos and neutralinos. Dirac masses $m_D$ owe their smallness to the pattern of light-heavy scales in the neutralino mass matrix. The smallness of the Majorana masses $m_L$ is linked to a similar seesaw pattern in the sneutrino mass matrix. Two distinct scenarios emerge. In the first, with very small or vanishing $M_R$, the physical neutrino eigenstates are, for each generation, either two light Majorana states with a mixing angle ranging from very small to maximal, depending on the ratio $m_D/M_R$, or one light Dirac state. In the second scenario, with a large value of $M_R$, the physical eigenstates are two nearly unmixed Majorana states with masses $\sim m_L$ and $\sim M_R$. In both cases, the $(B-L)$-breaking scale $M_G$ is, in general, much smaller than that in the traditional seesaw mechanism.

## I. INTRODUCTION

It has long been known that neutrinos are very light. Whether they are only light or completely massless, however, is a question which has remained unanswered up until recently.

No direct measurement of neutrino masses exists as yet. Nevertheless, overwhelming evidence has been gathering in the past years of the fact that neutrinos of different generations oscillate one into the other [1]. Oscillations have been observed by atmospheric [3], solar [4], and accelerator [5] neutrino experiments. Quantum mechanics relates oscillations among neutrinos of different flavor, $\nu_e$, $\nu_\mu$, $\nu_\tau$, to nonvanishing values of mass for the neutrino mass eigenstates, $\nu_1$, $\nu_2$, and $\nu_3$, which are linear combinations of $\nu_e$, $\nu_\mu$, and $\nu_\tau$. The profusion of experimental results on neutrino oscillations turns, therefore, into a strong indication that neutrinos have mass.

This fact has profound consequences for particle physics, astrophysics, and cosmology. Having mass, neutrinos may be a crucial ingredient of the hot dark matter component that contributes to the mass density of the Universe [6]. Moreover, if their mass does not exceed the eV range, they may have played an important role in the formation of the observed structure of galaxies [6]. Far from being a tested result, this value is a constraint to be kept into account by model building, more stringent than the direct bounds from collider experiments: $m_{\nu_e} \lesssim 5\,\mathrm{eV}$, $m_{\nu_\mu} \lesssim 170\,\mathrm{keV}$, and $m_{\nu_\tau} \lesssim 18\,\mathrm{MeV}$ [2]. It is also possible, however, that some or all neutrinos are even lighter than a few eV. Indeed, oscillation experiments give only an indication on the splitting of neutrino masses squared, but leave the absolute size of neutrino masses still open to speculation. They are not yet conclusive even on the number of neutrino species undergoing oscillations.

The experimental situation can be schematically summarized as follows. Atmospheric neutrino experiments from Super-Kamiokande [3,7] indicate that muon neutrinos $\nu_\mu$'s oscillate into different neutrinos, $\nu_x$'s. The mass-squared splitting of the two mass eigenstate neutrinos (whose main components are $\nu_\mu$ and $\nu_x$) is $10^{-3}\,\mathrm{eV}^2 < \Delta m^2 < 10^{-2}\,\mathrm{eV}^2$. The neutrino $\nu_x$ seems to be predominantly a tau neutrino $\nu_\tau$ [7]. Results from the CHOOZ reactor experiment [8] strongly suggest that $\nu_x$ is unlikely to be the electron neutrino $\nu_e$, although this possibility cannot be excluded by the Super-Kamiokande Collaboration [7]. Solar neutrino experiments [4] indicate that $\nu_e$'s oscillate into some other type of neutrinos $\nu_y$'s. If the MSW mechanism [9] is used to explain the deficit in the solar neutrino flux, the relevant mass-squared splitting is $\Delta m^2 \sim 10^{-5}\,\mathrm{eV}^2$ for the large and small mixing angle solution, or $\Delta m^2 \sim 10^{-7}\,\mathrm{eV}^2$ for the LOW solution [10]. It is $\Delta m^2 \sim 10^{-10}\,\mathrm{eV}^2$ if oscillations in the vacuum between Sun and Earth [11] are invoked. Finally, the Liquid Scintillator Neutrino Detector (LSND) accelerator experiment [5] seems to support the hypothesis of a $\nu_\mu \to \nu_e$ oscillation with corresponding mass-squared splitting $\Delta m^2 \sim 1\,\mathrm{eV}^2$.

Barring experimental systematic errors which may lead to inconsistent results, these data cannot be explained by assuming the existence of only three light neutrinos and that each of these oscillations is described by a single $\Delta m^2$ [1]. One simple way out is to assume that there exist additional neutrinos, also light. Accurate measurements of the invisible width of the $Z$ boson at the CERN $e+e-$ collider LEP have unequivocally shown that there are only three light neutrinos coupling to the $Z$ boson [12]. Other possible light states, therefore, must have as main constituent neutrinos invisible for the $Z$ boson, i.e., right-handed neutrinos. When discussing neutrino oscillations, these are often called sterile neutrinos, in contrast to the left-handed components of SU(2) doublets, or active neutrinos. If sterile neutrinos exist, the results of solar neutrino experiments may be viewed in terms of oscillations between electron and sterile neutrinos. Up until recently, these were preferably accommodated in a four-neutrino spectrum of the "$2+2$" variety. This spectrum consists of two pairs of neutrinos separated by the LSND $\Delta m^2$, with the component of each pair split, respectively, by the atmospheric and solar $\Delta m^2$. Recent measurements of the solar neutrino flux by the Super-Kamiokande Collaboration, however, seem to disfavor the possibility of $\nu_y$'s being sterile neutrinos at the 95% C.L. [13]. (A different conclusion is reached in [14].) This fact, together with a lower oscillation probability recently claimed by LSND [15], resurrects the "3+1" neutrino spectrum [16] previously excluded by the incompatibility between LSND and negative searches for $\nu_e$ and $\nu_\mu$ disappearance. In this spectrum, one mainly-sterile neutrino is separated by the LSND mass gap from the other three, which are mainly active, roughly degenerate, and sufficient to explain atmospheric and solar neutrino data. The "$3+1$" spectrum assumes a small mixing between the active and sterile content of the four mass eigenstates. For a model in which this spectrum can be easily implemented, see Ref. [17].

The challenge, which particle physics is then faced with, is twofold. On one side there is the longstanding issue of the smallness of neutrino masses. Although clear experimental evidence that neutrinos are massive is a recent acquisition, there has always been a strong theoretical prejudice in this direction. The exact vanishing of neutrino masses, which is consistent with the standard-model (SM) particle content, would nevertheless demand for a deeper explanation, possibly through a symmetry principle, in an underlying theory of which the SM is an effective limit. Rejecting the possibility of vanishing neutrino masses on theoretical ground, there has been a tremendous effort to explain why these masses are much smaller than those of quarks and charged leptons.

Technically, two ways are known to obtain light neutrinos. The first follows the conventional mechanism for generating quark and charged-lepton masses. The SM is enlarged to include three right-handed neutrinos. These couple to the neutral components of the left-handed leptonic doublets through Yukawa interaction terms. After electroweak symmetry breaking, there remain three light mass eigenstates $\nu_1$, $\nu_2$, and $\nu_3$, which are Dirac fermions. For these neutrinos to be at the eV scale, Yukawa couplings as small as $\sim 10^{-11}$ are needed. The second known way of generating small neutrino masses is through the seesaw mechanism [18,19]. This invokes the existence of right-handed neutrinos with a very large mass $M_R \sim 10^{12}\,\mathrm{GeV}$, whereas the couplings of the Yukawa neutrino interactions are unsuppressed. The smallness of the light neutrino masses is then explained in terms of the seesaw pattern of light-heavy scales in the neutrino mass matrix. The three light mass eigenstates $\nu_1$, $\nu_2$, and $\nu_3$ are now Majorana fermions, with mass $\sim m^2_{\mathrm{weak}}/M_R$. The other three mass eigenstates, $n_1$, $n_2$, and $n_3$, also of Majorana type, remain at the high scale $M_R$. (On a different line of thought is the recent proposal of Ref. [20], where the smallness of neutrino masses is explained by embedding the SM and right-handed neutrinos in a higher-dimensional spacetime. In a scenario of Dirac neutrino masses, for example, the Dirac Yukawa couplings between left-handed neutrinos localized on the SM three-brane and the right-handed ones, residing in the bulk, are suppressed by the volume factor of compactified dimensions.)

The other challenge for theoretical particle physics has been unexpectedly kindled by recent experimental results. The interpretation of data coming from solar neutrino experiments in terms of oscillations between active and sterile neutrinos requires extensions of the SM that explain the existence of light mass eigenstates, other than $\nu_1$, $\nu_2$, $\nu_3$, mainly corresponding to additional SU(2)× U(1) singlets. It is possible that the experimental situation will evolve in the future in such a way that (i) the preliminary findings of the Super-Kamiokande Collaboration on solar neutrino oscillations disfavoring the sterile neutrino option are confirmed, (ii) the LSND results are clearly excluded. The SNO experiment [21] will soon be able to resolve the issue as to whether the electron neutrinos produced in the Sun oscillate to active or sterile neutrinos. In the meantime, the Super-Kamiokande Collaboration will keep collecting more data on solar neutrino oscillations that may possibly strengthen their preliminary results. Moreover, there is a number of already operating [22] and planned [23] accelerator experiments to seek for oscillations in the LSND region. If no signal is found in these searches, the two $\Delta m^2$ of solar and atmospheric neutrino oscillations can be easily accommodated in a scenario of three light neutrinos. This, of course, does not mean that sterile neutrinos do not exist. It may only mean that low-energy physics is blind to them, either because they are heavy or because, even if light, they are very weakly mixed with the active ones. Ideal detectors may turn out to be astronomical ones, such as supernovas. Sterile neutrinos with a mass of a few keV (or tens of keV) have been proposed to explain the motion of pulsars born in supernova explosions [24]. Light sterile neutrinos, with mass above the eV scale, may also be of interest in explaining supernova nucleosynthesis and big bang nucleosynthesis [25]. If in the mass range of 0.1 keV, they may also give rise to "cool" dark matter, as recently discussed in [26].

The natural setting for incorporating sterile neutrinos is that in which six final Majorana states are seesaw generated. Nevertheless, no satisfactory theoretical model has been found so far to induce the suppression factor $\sim m^2_{\rm weak}/M_R$ needed to have light $\nu_1$, $\nu_2$, and $\nu_3$ states, while simultaneously keeping very low the scale for some or all the right-handed neutrinos, with resulting light states among $n_1$, $n_2$, and $n_3$. (Infinitely many light sterile neutrinos, however, are obtained in extra-dimensional mechanisms for generating light neutrino masses [20].) In a typical model, the seesaw right-handed neutrinos may participate in an additional gauge interaction spontaneously broken at some scale $M_G$. In the simplest known model, free from gauge anomalies and mixed gauge and gravitational anomalies, this new interaction is $U(1)_{B-L}$ and the SM particle content is minimally enlarged by a neutral gauge boson, three additional electroweak singlets, and a neutral $(B-L)$-breaking Higgs boson. The mass of these new particles is closely related to the spontaneous violation of $B-L$ and all these particles are at the large scale $M_G$ ($\sim M_R$).

It is in general assumed that similar features hold also in the supersymmetric version of such a model. In this paper we explore other possibilities that naturally present themselves in supersymmetric models, in addition to the two outlined above, of generating small neutrino masses. Two mechanisms are described. In both, lowest-order Yukawa operators are forbidden by imposing some horizontal symmetries. Small neutrino masses can then be generated at the tree level, through higher-dimensional nonrenormalizable operators, in the first mechanism, or radiatively, through the virtual exchange of superpartners, in the second one. The first mechanism, described in Sec. II, does not require an additional $U(1)_{B-L}$ gauge interaction. The gauging of $U(1)_{B-L}$ is not needed either for the radiative generation of Majorana masses for active neutrinos, but it is, on the contrary, necessary to obtain radiative Dirac neutrino masses. For the sake of a simpler and more coherent discussion, however, the radiative mechanism is presented in the case of a gauged $U(1)_{B-L}$ symmetry. The modifications to the case without a $U(1)_{B-L}$ symmetry, when applicable, are obvious and will be briefly commented upon. The radiative mechanism is introduced in Sec. III and worked out in Secs. IV, V and VI. The patterns of physical neutrino masses that can arise through both mechanisms are classified in Sec. VII. A possible embedding of these two mechanisms in full-fledged models is outlined in Sec. VIII. Up to this section, it is implicitly or explicitly assumed that the signal of supersymmetry breaking is transmitted from the hidden to the visible sector through supergravity mediation. Comments will be made in Sec. VIII on possible changes induced when this transmission is realized through gauge mediation. The paper ends with some final conclusions and remarks in Sec. IX.

## II. SUPERSYMMETRIC TREE-LEVEL MECHANISM

The class of models considered in this paper [1] has three SU(2)× U(1) singlet superfields $\bar{N}$ in addition to the three lepton doublets $L$, with neutral components $N$. The lowest-order operators giving rise to neutrino masses are

$$W = \frac{1}{4}\frac{h_\nu}{M_P} LHLH + y_\nu \bar{N}LH + \frac{1}{2}M_R\bar{N}\bar{N}\,, \tag{1}$$

with the scale $M_R$ naturally given by the Planck mass $M_P$. All these operators can be forbidden by a horizontal discrete symmetry $Z_n$ in conjunction with a continuous or discrete symmetry such as lepton number. As for the first symmetry, it is sufficient to assign vanishing $Z_n$ charges to the superfields $L$ and $H$ and $Z_n(\bar{N}) = +1$ to the superfields $\bar{N}$. This assignment forbids the lowest-order Yukawa operators, but leaves allowed higher-dimensional operators of Yukawa type,

$$W = \frac{k_\nu}{M_P}\, Z\bar{N}LH\,, \tag{2}$$

where $Z$ is a spurion superfield with charge $Z_n(Z) = -1$. The second symmetry (different from $Z_2$) assigns the following charges to the lepton superfields:

$$L(+1), \quad \bar{N}(-1), \quad \bar{E}(-1), \tag{3}$$

and zero charge to all the other superfields. Therefore, it forbids both Majorana mass operators in Eq. (1) as well as the higher-dimensional operators:

[1] Tree-level mechanisms were also advocated in [27] in the context of superstring theories, in [28] in the context of supersymmetric models with gauge mediation of supersymmetry breaking, and in [29] in the context of supergravity models.

$$W = \frac{1}{4}\frac{z_\nu}{M_P} ZZ\bar{N}\bar{N}\,. \tag{4}$$

In this scenario, the emerging picture is that of three very light neutrino states of Dirac type. The spurion field $Z$ acquires, in general, a supersymmetry-conserving vacuum expectation value (VEV) $\mathcal{A}_Z$ and a supersymmetry-violating one $\mathcal{F}_Z$. Through operators analogous to that in Eq. (2), the VEV $\mathcal{A}_Z$ gives contributions to Yukawa couplings for quarks and charged leptons that are negligibly small, unless $\mathcal{A}_Z$ is at some large scale $\gtrsim 10^{13}$ GeV. Acceptable Yukawa couplings may, however, be generated for neutrinos. Indeed, in a class of O'Raifeartaigh-type models discussed in Sec. VIII, $\mathcal{A}_Z$ is vanishing in the supersymmetric limit, but is shifted to a value

$$\mathcal{A}_Z \simeq \frac{16\pi^2}{\lambda^3} m_{\rm weak} \tag{5}$$

by supersymmetry-breaking effects, where $\lambda$ is a dimensionless coupling. The induced Yukawa couplings

$$y_\nu = 16\pi^2\, \frac{k_\nu}{\lambda^3}\frac{m_{\rm weak}}{M_P} \tag{6}$$

give rise to Dirac neutrino masses of order $10^{-2}$ eV if $\lambda$ and $k_\nu$ are of $\mathcal{O}(1)$, [2] i.e., in the range of the $\Delta m^2$ required by the MSW solution to the solar neutrino deficit. Smaller values of $\lambda$ allow also neutrino masses in the range of atmospheric neutrino experiments. No room is left for sterile neutrinos and the observed oscillation patterns can be explained through flavor oscillations. This tree-level mechanism for generating small neutrino Yukawa couplings is rather generic, in the sense that it can be easily incorporated in any supersymmetric extension of the SM model.

It is also possible to forbid the renormalizable operators in Eq. (1) leaving, however, allowed both operators in Eqs. (2) and (4), as well as the nonrenormalizable operator in (1). One way to achieve this is to couple the above $Z_n$ symmetry with an $R$-parity symmetry under which $\bar{N}$ is odd and $Z$ is even. Dangerous terms of type $M_P Z\bar{N}$ are then forbidden, while Majorana mass terms $\sim M_P\bar{N}\bar{N}$ are avoided by requiring that $Z_n$ is not $Z_2$. In this case, both Dirac and Majorana masses become possible at the tree level: $\sim \mathcal{A}_Z v/M_P$ is obtained for Dirac masses; $\sim \mathcal{A}_Z^2/M_P$ and $\sim v^2/M_P$, for Majorana masses for right- and left-handed neutrinos, respectively, where $v$ is the VEV of the neutral component of $H$. The physical neutrino states are six, of Majorana type: the three $n_i$ states have masses $\sim \mathcal{A}_Z^2/M_P$, the three states $\nu_i$ are lighter, i.e., of order of the Majorana mass for left-handed neutrinos, $\sim v^2/M_P$. In the class of models in which supersymmetry breaking induces the value of $\mathcal{A}_Z$ in Eq. (5), these masses range from the keV to the sub-eV region, depending on the value of $\lambda$ and of the other dimensionless couplings $z_\nu$, $h_\nu$, and $k_\nu$. Thus, light sterile neutrinos can be easily accommodated in such a scenario.

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[2] We thank T. Yanagida for pointing this out to us.

It should be noticed that the operator (4) gives rise to bilinear soft supersymmetry-breaking terms with large massive couplings, $BM_R \sim \mathcal{A}_Z \mathcal{F}_Z / M_P$. Such terms may induce VEV's for the scalar components of the superfields $\bar{N}$ if $\mathcal{F}_Z$ is of the same order of the largest supersymmetry-breaking VEV $\sim m_{\rm weak} M_P$. A value of $\mathcal{F}_Z$ low enough to avoid the spontaneous breaking of lepton number decreases also the value of the trilinear parameters $A \sim \mathcal{F}_Z / M_P$ generated by the operator (2). Nevertheless, large radiative contributions to the Majorana mass for active neutrinos arise. We discuss these contributions explicitly in the case of a gauged $U(1)_{B-L}$, when also potentially large contributions to Dirac masses are induced.

## III. SUPERSYMMETRIC RADIATIVE MECHANISM

If the gauge sector is enlarged as in a typical model that leads to the seesaw mechanism, the phenomenology of the neutrino sector becomes much richer. The phenomenology of other sectors becomes also more interesting. The presence of the heavy scale $M_G$, typical of seesaw models, for example, is felt also in other sectors. Some neutralinos become very heavy and, depending on the details of the specific realization of the model, also some supersymmetric partners of neutrinos, or sneutrinos, can be at the same large scale of the $B-L$ violation.

A suppression factor for small neutrino masses, therefore, does not necessarily need to be induced by the right-handed neutrinos. Indeed, there is at least one and, possibly, two mass matrices (neutralino and sneutrino mass matrices) with the same seesaw pattern of light-heavy scales that was only present in the neutrino mass matrix in the nonsupersymmetric case. It is clear that the transmission to the neutrino sector of a suppression factor induced in the neutralino and/or sneutrino sector, may only happen at the quantum level. [3] The advantage of such a procedure is obvious: the mechanism of mass generation for the light neutrinos can be disentangled from the scale of the sterile neutrinos, which does not need to be anymore that of the $B-L$ violation.

If a program of radiative generation of neutrino mass operators is implemented, the Yukawa couplings between active and sterile neutrinos become redundant. As in the generic supersymmetric scenario outlined above, they can be altogether forbidden. Higher-dimension operators of Yukawa type [see Eq. (2)] remain in general allowed and provide Dirac neutrino masses naturally of order $10^{-2}$ eV. If larger physical masses are aimed at–for example, in the eV range–then the largest contribution to the Dirac neutrino mass can be of radiative origin, if the scale of $B-L$ breaking is not too large. If this is not the case, both tree-level and radiative mechanisms complement each other, providing comparable

[3]Neutrino masses can be obtained radiatively also in other models. Among them, the Zee model and its supersymmetric version have long been known. For recent discussions on this possibility and references to other radiative mechanism scenarios, see Ref. [30]. See also Ref. [31] where Dirac masses are obtained at the quantum level in nonsupersymmetric models with extended gauge groups. Very popular are also $R_p$-violating models in which two neutrino masses are generated radiatively [32].

contributions. The radiative contribution to the Majorana mass for active neutrinos is, on the contrary, always larger than the tree-level contribution arising in the second tree-level scenario described in Sec. II. The ingredients necessary for generating radiatively neutrino mass operators can be supplied by soft supersymmetry-breaking terms. The chiral-flavor violation needed for a Dirac mass operator can be provided by trilinear soft terms [33,34]. For a Majorana mass operator, the correct SU(2) structure can also be induced by the same trilinear terms, whereas the explicit lepton-number violation can come from bilinear soft terms in the scalar components of the singlets $\bar{N}$. In both cases, the correct number of $R$ charges for Dirac and Majorana mass operators, is provided by neutralino mass terms.

The supersymmetric models that we consider here contain a $B-L$ symmetry broken at some large scale $M_G$ and three additional SM singlets $\bar{N}$ with tree-level mass $M_R$. Dirac mass operators mixing active and sterile neutrinos as well as Majorana mass operators for active neutrinos are generated at the quantum level via loops with virtual exchange of neutralinos and sneutrinos. Explicit expressions for the corresponding couplings $m_D$ and $m_L$ are given in Sec. V after a detailed description of the neutralino and sneutrino mass matrices in Sec. IV. For unsuppressed soft parameters $A$ and $B$ ($A \sim B \sim m_{\rm weak}$), Dirac masses $m_D$ are of order $\sim (m_{\rm weak}/8\pi^2)(m_{\rm weak}/M_G)^2$, whereas Majorana ones are $\sim (m_{\rm weak}/8\pi^2)(m_{\rm weak}/M_R)^3$ for $M_R \gg m_{\rm weak}$ and $\sim (BM_R/m_{\rm weak}/8\pi^2)$ for $M_R \ll m_{\rm weak}$. (The qualitative behavior of the radiative contribution to Majorana masses obtained in the second scenario of Sec. II is discussed in Sec. V.) Of the resulting six eigenvalues, three are certainly light. The other three are light or heavy depending on the value of the tree-level Majorana masses $M_R$ for the singlets $\bar{N}$.

Thus, two classes of possible scenarios emerge, discriminated by the value of the mass parameter $M_R$. In the first one $M_R$ is large, say, $M_R \sim M_G$, and the resulting three states $n_i$ are of $\mathcal{O}(M_G)$. The states $\nu_i$ can be easily of $\mathcal{O}$(eV) or less. The main contribution to the corresponding eigenvalues comes from the Majorana masses $m_L$, which dominate over $m_D^2/M_R$. Therefore, the heavy $U(1)_{B-L}$ scale can be as light as a few hundred TeV. [Notice that, if the lack of a dynamical explanation for such a large scale $M_R$ is tolerated, the same neutrino spectrum can be obtained without introducing a $U(1)_{B-L}$ symmetry.]

In the second class, $M_R$ is very light and Dirac masses play a much more relevant role. The $n_i$ states can be as light as, or lighter than, the $\nu_i$'s, which are at the eV scale if $M_G$ is $\sim 10^7$ GeV. They can be heavier, as, for example, up to the scale required to explain the motion of pulsars. They can also be degenerate with the $\nu_i$ states in the particular case of the exact vanishing of $M_R$: in this case, for each generation $i$, the two Majorana states $\nu_i$ and $n_i$ are equivalent to one Dirac neutrino. The key ingredients for this second class of scenarios are the lightness of $M_R$ and the seesaw suppression of $m_D$ originating in the neutralino mass matrix. Ways to render $M_R$ much smaller than $M_G$ are discussed in Sec. IV.

From the phenomenological point of view, $U(1)_{B-L}$ scales as low as those needed for these two classes of scenarios are either too heavy or marginally suited to give clear collider signatures due to the additional neutral gauge boson or, possibly, heavy $\bar{N}$ states. Drastic deviations from the phenomenology of the minimal supersymmetric standard model may, however, be observed in the scalar sector if $M_R \lesssim m_{\rm weak}$. Phenomenological consequences as well as cosmological implications of these two scenarios are postponed to later work. Finally, in Sec. VI, we recall that in a scenario of radiative neutrino masses, also Yukawa couplings are generated radiatively. These are now *naturally* very small: i.e., $\sim m_D/m_{\rm weak}$ for the dimensionless Dirac coupling and $m_L/m_{\rm weak}^2$ for the dimensionful Majorana coupling. For Dirac and Majorana masses in the eV range, they are in general larger than the coupling induced by Yukawa operators of higher dimensionality.

## IV. RADIATIVE SEESAW MECHANISMS

Six massive Majorana neutrino states are usually obtained in a three-family scenario with three left-handed neutrinos $\nu_L$ and three right-handed ones $\nu_R$ with mass operators

$$-\mathcal{L}_{\rm mass} = -\frac{1}{2}\nu_L^T C\, m_L\, \nu_L + \bar{\nu}_R\, m_D\, \nu_L - \frac{1}{2}\nu_R^T\, C\, M_R^*\, \nu_R + {\rm h.c.}\,, \tag{7}$$

where $C$ is the charge conjugation matrix. Here and in the following, all family indices are omitted for simplicity. In the conventional seesaw mechanism, with $m_D \simeq m_{\rm weak} \ll M_R$, the six Majorana states are split into three light ones, $\nu \simeq \nu_L + \nu_L^c$, with mass $m_\nu \simeq -m_D\,(1/M_R)\, m_D^T$ and three heavy ones, $n \simeq \nu_R + \nu_R^c$, with mass $m_n \simeq M_R$.

In supersymmetric models with violation of $B-L$ at some large scale $M_G$, both Dirac and Majorana mass operators can be generated radiatively through neutralino-sneutrino loops. Suitable suppression factors, with the task of keeping $m_D$ and $m_L$ well below the electroweak scale, can be induced by the seesaw pattern of high-low scales in the neutralino and/or sneutrino mass matrix.

The SM fields are charged under the $B-L$ symmetry, which, for simplicity, is assumed to be a $U(1)$ symmetry. Nevertheless, the following discussion can be easily generalized to larger gauge groups. Under $U(1)_{B-L}$, quark, lepton, and Higgs chiral superfields are charged in such a way that all Yukawa interaction terms are invariant under $U(1)_{B-L}$. A charge assignment that leads to vanishing gauge anomalies is, however, unique [35]. It is given by

$$\begin{gathered} X_Q = +1,\;\; X_{\bar U} = +1,\;\; X_{\bar D} = -3,\;\; X_H = -2,\;\; X_{\bar H} = +2, \\ X_L = -3,\;\; X_{\bar E} = +1,\;\; X_{\bar N} = +5. \end{gathered} \tag{8}$$

Neutrino Yukawa terms such as those in Eq. (1) can be forbidden at the tree level by the discrete $Z_n$ symmetry discussed in Sec. II. On the other hand, soft supersymmetry-breaking trilinear terms involving sneutrino fields can be induced by superpotential terms such as those in Eq. (2) when the spurion field $Z$, which is neutral under $U(1)_{B-L}$ and has $Z_n$ charge $Z_n(Z) = -1$, acquires the supersymmetry-violating VEV $\mathcal{F}_Z$. As already explained in Sec. II, the same operator gives rise to tree-level Yukawa couplings when $Z$ acquires the supersymmetry-conserving VEV $\mathcal{A}_Z$. In order to avoid too large neutrino masses, a hierarchy between $\mathcal{A}_Z$ and $\mathcal{F}_Z$ must exist. The possibility of implementing such a hierarchy, which was already assumed in Sec. II, will be discussed in Sec. VIII. The tree-level contributions can then be naturally suppressed to give neutrino Yukawa couplings as small as $\sim 10^{-13}$ and Dirac masses well below the eV range.

Majorana mass terms $(1/2)M_R\bar{N}\bar{N}$ are assumed to exist at the tree level. It is easy to see how $M_R$ can be completely disentangled from $M_G$. Majorana mass terms, indeed, are induced by higher-dimensional operators in the superpotential [(different from those considered in the case without a $U(1)_{B-L}$ symmetry discussed in Sec. II] that are neutral under $Z_n$ and $U(1)_{B-L}$. For a generic $Z_n$ symmetry, these operators are

$$W = \frac{1}{4}\frac{1}{(M_P)^{m+1}}\,\bar{\Phi}^m ZZ\bar{N}\bar{N}\,, \tag{9}$$

whereas, if the discrete symmetry $Z_n$ is $Z_2$, the lowest-order operators are

$$W = \frac{1}{2}\frac{1}{(M_P)^{m-1}}\,\bar{\Phi}^m \bar{N}\bar{N}\,. \tag{10}$$

In both cases, the field $\bar{\Phi}$ is a SM singlet that breaks $U(1)_{B-L}$ and gets a VEV of $\mathcal{O}(M_G)$. It is charged under $U(1)_{B-L}$ with charge $X_{\bar{\Phi}}$. The power $m$ in Eqs. (9) and (10) depends on $X_{\bar{\Phi}}$, once the charge assignment for $\bar{N}$ is fixed as in Eq. (8). It is clear, then, that the Majorana mass $M_R$ does not need to be of the same order of $M_G$, and it can get values over a wide range of scales below $M_G$. Indeed, if the discrete symmetry that forbids tree-level Yukawa terms is $Z_2$, it may be of $\mathcal{O}(M_G)$ if $m = 1$. If $m = 2$, depending on the value of $M_G$, it may be a few order of magnitude above the electroweak scale or at the eV scale (see possible values of $M_G$ discussed in Sec. VII). It can also be extraordinarily suppressed if a generic $Z_n$ symmetry is assumed and if $\bar{\Phi}$ has the $U(1)_{B-L}$ charge $X_{\bar{\Phi}} = -1$. For the charge assignment in Eq. (8), it is, for example, $M_R \sim M_G^{10}\,\mathcal{A}_Z^2/(M_P)^{11}$. Finally, if we impose the symmetry in Eq. (3), the higher dimensional operators in Eqs. (9) and (10) are forbidden and the exact vanishing of $M_R$ is guaranteed. [4]

Notice how the presence of a gauged $U(1)_{B-L}$ symmetry prevents, in general, unwanted vacua with non-vanishing lepton number. Indeed, in contrast to the situation described in the second scenario of Sec. II, no dangerously large bilinear soft supersymmetry-breaking terms $BM_R$ are induced by the same operators that generate $M_R$.

Tree-level Majorana mass terms for left-handed neutrinos are strongly suppressed. They may arise from superpotential operators:

$$W = \frac{1}{4}\frac{1}{(M_P)^{m+1}}\,\Phi^m LHLH\,, \tag{11}$$

where $\Phi$ is another SM singlet needed in addition to $\bar{\Phi}$ to cancel $U(1)_{B-L}$ anomalies. It has $U(1)_{B-L}$ charge opposite to that of $\bar{\Phi}$. As in the case of the operators in Eqs. (9) and (10), also these Majorana mass terms can be forbidden by the symmetry in Eq. (3).

[4] Depending on $X_{\bar{\Phi}}$, it is also possible that no Majorana mass term for $\bar{N}$ is allowed due to a residual unbroken discrete subgroup of $U(1)_{B-L}$, even if no other symmetry as that in Eq. (3) is imposed.

### A. Sneutrino mass matrix

As long as $M_R$ is different from zero, a bilinear and lepton number-violating soft supersymmetry-breaking term $(1/2)BM_R\tilde{\bar{N}}\tilde{\bar{N}}$ is always induced. Since the parameter $B$ is, in general, at the electroweak scale, this term may be very large or very small, depending on the value of $M_R$. The complete sneutrino mass potential is given by

$$\begin{aligned} -\mathcal{L}_{\rm mass} &= m_{\tilde{l}}^2\,\tilde{N}^*\tilde{N} + m_{\tilde{\nu}}^2\,\tilde{\bar{N}}^*\tilde{\bar{N}} + M_R^2\,\tilde{\bar{N}}^*\tilde{\bar{N}} + \frac{1}{2}M_R\left(B\tilde{\bar{N}}\tilde{\bar{N}} + B^*\tilde{\bar{N}}^*\tilde{\bar{N}}^*\right) \\ &\quad + \left(Av\tilde{N}\tilde{\bar{N}} + A^*v\tilde{N}^*\tilde{\bar{N}}^*\right)\,, \end{aligned} \tag{12}$$

where $v$ is the VEV of the relevant Higgs doublet. When redefining the sneutrino fields as $\tilde{\nu}_L \equiv \tilde{N}$ and $\tilde{\nu}_R \equiv \tilde{\bar{N}}^*$, the above expression for $-\mathcal{L}_{\rm mass}$ becomes

$$\begin{aligned} -\mathcal{L}_{\rm mass} &= m_{\tilde{l}}^2\,\tilde{\nu}_L^*\tilde{\nu}_L + m_{\tilde{\nu}}^2\,\tilde{\nu}_R^*\,\tilde{\nu}_R + M_R^2\,\tilde{\nu}_R^*\,\tilde{\nu}_R + \frac{1}{2}M_R\left(B\,\tilde{\nu}_R^*\,\tilde{\nu}_R^* + B^*\tilde{\nu}_R\tilde{\nu}_R\right) \\ &\quad + \left(Av\,\tilde{\nu}_R^*\,\tilde{\nu}_L + A^*v\,\tilde{\nu}_L^*\tilde{\nu}_R\right)\,. \end{aligned} \tag{13}$$

A nonvanishing soft parameter $B$ in Eq. (13) induces a splitting in the $CP$-even and $CP$-odd sneutrino states. By decomposing $\tilde{\nu}_L$ and $\tilde{\nu}_R$ as $\tilde{\nu}_L \equiv (\tilde{\nu}_{L+} + i\tilde{\nu}_{L-})/\sqrt{2}$ and $\tilde{\nu}_R \equiv (\tilde{\nu}_{R+} + i\tilde{\nu}_{R-})/\sqrt{2}$, the sneutrino mass potential can, then, be expressed as

$$-\mathcal{L}_{\rm mass} = \frac{1}{2}\left(\,\tilde{\nu}_{L+}\ \ \tilde{\nu}_{R+}\ \ \tilde{\nu}_{L-}\ \ \tilde{\nu}_{R-}\,\right)\mathcal{M}^2_{\rm sneut}\begin{pmatrix}\tilde{\nu}_{L+}\\ \tilde{\nu}_{R+}\\ \tilde{\nu}_{L-}\\ \tilde{\nu}_{R-}\end{pmatrix}\,. \tag{14}$$

The matrix $\mathcal{M}^2_{\rm sneut}$ is, in general, a rather complicated matrix. For one generation neutrino fields, i.e., ignoring the flavor-changing structure of $A$, $B$, $m_{\tilde{l}}$, and $m_{\tilde{\nu}}$, $\mathcal{M}^2_{\rm sneut}$ is the following $4\times 4$ matrix:

$$\left(\begin{array}{cc|cc} m_{\tilde{l}}^2 & \frac{1}{2}(A{+}A^*)\,v & 0 & \frac{i}{2}(A^*{-}A)\,v \\ \frac{1}{2}(A{+}A^*)\,v & \left(M_R^2{+}\frac{1}{2}(B{+}B^*)M_R{+}m_{\tilde{\nu}}^2\right) & \frac{i}{2}(A{-}A^*)\,v & \frac{i}{2}(B^*{-}B)M_R \\ \hline 0 & \frac{i}{2}(A{-}A^*)\,v & m_{\tilde{l}}^2 & \frac{1}{2}(A{+}A^*)\,v \\ \frac{i}{2}(A^*{-}A)\,v & \frac{i}{2}(B^*{-}B)M_R & \frac{1}{2}(A{+}A^*)\,v & \left(M_R^2{-}\frac{1}{2}(B{+}B^*)M_R + m_{\tilde{\nu}}^2\right) \end{array}\right)\,. \tag{15}$$

This matrix simplifies drastically when $A$ and $B$ are made real using phase rotations of the $N$ and $\bar{N}$ superfields: it reduces, in this case, to the block diagonal matrix

$$\mathcal{M}^2_{\rm sneut} = \begin{pmatrix}\mathcal{M}^2_+ & 0_2\\ 0_2 & \mathcal{M}^2_-\end{pmatrix}\,;\qquad \text{with}\quad \mathcal{M}^2_\pm = \begin{pmatrix} m_{\tilde{l}}^2 & Av\\ Av & M_R^2 \pm BM_R + m_{\tilde{\nu}}^2\end{pmatrix}\,. \tag{16}$$

(The same decomposition of $\mathcal{M}^2_{\rm sneut}$ is found in Ref. [36]; see also [37].)

For large $M_R$ ($\sim M_G$), the two matrices $\mathcal{M}^2_\pm$ have the typical pattern of light-heavy scales present in the neutrino mass matrix in the conventional seesaw mechanism. Of the four eigenstates, $\tilde{\nu}_{+,1}$, $\tilde{\nu}_{+,2}$, $\tilde{\nu}_{-,1}$, and $\tilde{\nu}_{-,2}$, two ($\tilde{\nu}_{\pm,1}$) are at the electroweak scale and two ($\tilde{\nu}_{\pm,2}$) are heavy, i.e., at the scale $M_R$. The two states in the light pair and those in the heavy pair have masses that are split by terms proportional to $B$. These splitting terms are suppressed by powers of $(m_{\rm weak}/M_R)$. It is, indeed

$$\begin{aligned} m^2_{\tilde{\nu}_{\pm,1}} &= m^2_{\tilde{l}}\left\{1-(ac)^2\beta^2 \pm (ac)^2 b\beta^3 \ + \ \mathcal{O}(\beta^4)\right\} \\ m^2_{\tilde{\nu}_{\pm,2}} &= M^2_R\left\{1 \pm b\beta + d^2\beta^2 \ + \ \mathcal{O}(\beta^4)\right\} , \end{aligned} \tag{17}$$

where the definitions

$$\beta \equiv \frac{m_{\tilde{l}}}{M_R}, \quad a \equiv \frac{A}{m_{\tilde{l}}}, \quad b \equiv \frac{B}{m_{\tilde{l}}}, \quad c \equiv \frac{v}{m_{\tilde{l}}}, \quad d \equiv \frac{m_{\tilde{\nu}}}{m_{\tilde{l}}}, \tag{18}$$

were used. At the same level of approximation, i.e., up to terms of $\mathcal{O}(\beta^4)$, the two diagonalization matrices of $\mathcal{M}^2_\pm$, $U_\pm$, are given by

$$\begin{pmatrix} \tilde{\nu}_{L\pm} \\ \tilde{\nu}_{R\pm} \end{pmatrix} = U_\pm \begin{pmatrix} \tilde{\nu}_{\pm,1} \\ \tilde{\nu}_{\pm,2} \end{pmatrix} \equiv \begin{pmatrix} 1 & -ac\left(-\beta^2 \pm b\beta^3\right) \\ ac\left(-\beta^2 \pm b\beta^3\right) & 1 \end{pmatrix} \begin{pmatrix} \tilde{\nu}_{\pm,1} \\ \tilde{\nu}_{\pm,2} \end{pmatrix} . \tag{19}$$

A Majorana mass operator radiatively induced by sneutrino-neutralino loops must have a coupling proportional to the mass splitting of the light or heavy sneutrino pairs, since this is induced by the lepton-violating $B$ parameter. Notice that the splitting of the two light eigenvalues comes only at order $(m_{\rm weak}/M_R)^3$. It is, therefore, conceivable that a Majorana mass term can be sufficiently suppressed by several powers of $(m_{\rm weak}/M_R)$.

For very tiny $M_R$ and small $(BM_R/m^2_{\rm weak})$, the correct expansion parameter needed to calculate eigenvalues and eigenstates of $\mathcal{M}^2_\pm$ is the inverse of $\beta$, which we indicate by $\gamma$. Since eigenvalues and eigenvectors are more involved in this case, we express them in terms of those obtained at the zeroth order in $\gamma$. For $M_R = 0$, $\mathcal{M}^2_+$ and $\mathcal{M}^2_-$ coincide, and the sneutrino mass potential has the well-known form

$$-\mathcal{L}_{\rm mass} = \begin{pmatrix} \tilde{\nu}^*_L & \tilde{\nu}^*_R \end{pmatrix} \begin{pmatrix} m^2_{\tilde{l}} & Av \\ Av & m^2_{\tilde{\nu}} \end{pmatrix} \begin{pmatrix} \tilde{\nu}_L \\ \tilde{\nu}_R \end{pmatrix} , \tag{20}$$

once the phase of the $A$ parameter is rotated away. Then, the eigenvalues are given by

$$m^2_{\tilde{\nu}_{1,2}} = \frac{1}{2}\left\{m^2_{\tilde{l}} + m^2_{\tilde{\nu}} \mp \sqrt{\left(m^2_{\tilde{l}} - m^2_{\tilde{\nu}}\right)^2 + 4\,(Av)^2}\right\}, \tag{21}$$

and the diagonalization matrix takes the following form:

$$U = \sqrt{\frac{m^2_{\tilde{\nu}} - m^2_{\tilde{\nu}_1}}{m^2_{\tilde{\nu}_2} - m^2_{\tilde{\nu}_1}}} \begin{pmatrix} 1 & \dfrac{Av}{m^2_{\tilde{\nu}} - m^2_{\tilde{\nu}_1}} \\ -\dfrac{Av}{m^2_{\tilde{\nu}} - m^2_{\tilde{\nu}_1}} & 1 \end{pmatrix} . \tag{22}$$

If $\gamma$ is nonzero, the masses for the $CP$-even and $CP$-odd sneutrino states split, and the eigenvalues of $\mathcal{M}^2_\pm$ now become

$$m^2_{\tilde{\nu}_{\pm,1}} = m^2_{\tilde{\nu}_1} \ \pm \ \frac{m^2_{\tilde{\nu}_2} - m^2_{\tilde{\nu}}}{m^2_{\tilde{\nu}_2} - m^2_{\tilde{\nu}_1}} m^2_{\tilde{l}}\, b\gamma + \mathcal{O}(m^2_{\tilde{l}}\gamma^2)\,,$$

$$m^2_{\tilde{\nu}_{\pm,2}} = m^2_{\tilde{\nu}_2} \ \pm \ \frac{m^2_{\tilde{\nu}} - m^2_{\tilde{\nu}_1}}{m^2_{\tilde{\nu}_2} - m^2_{\tilde{\nu}_1}} m^2_{\tilde{l}}\, b\gamma + \mathcal{O}(m^2_{\tilde{l}}\gamma^2)\,, \tag{23}$$

with $b$ defined in Eq. (18). At the same level of approximation, the corresponding diagonalization matrices $U_\pm$ are

$$U_\pm = \sqrt{\frac{m^2_{\tilde{\nu}} - m^2_{\tilde{\nu}_1}}{m^2_{\tilde{\nu}_2} - m^2_{\tilde{\nu}_1}}} \begin{pmatrix} 1 + x_\pm & \dfrac{Av}{m^2_{\tilde{\nu}} - m^2_{\tilde{\nu}_1}} - y_\pm \\ -\dfrac{Av}{m^2_{\tilde{\nu}} - m^2_{\tilde{\nu}_1}} + y_\pm & 1 + x_\pm \end{pmatrix}, \tag{24}$$

where $x_\pm$ and $y_\pm$ are given by

$$x_\pm = \pm \frac{(Av)^2}{\Big(m^2_{\tilde{\nu}} - m^2_{\tilde{\nu}_1}\Big)\Big(m^2_{\tilde{\nu}_2} - m^2_{\tilde{\nu}_1}\Big)^2}\, m^2_{\tilde{l}}\, b\gamma\,, \qquad y_\pm = x_\pm \left\{ \frac{Av}{m^2_{\tilde{\nu}} - m^2_{\tilde{\nu}_1}} + \frac{m^2_{\tilde{\nu}} - m^2_{\tilde{l}}}{Av} \right\}. \tag{25}$$

Differently than in the case with opposite hierarchy, i.e., $M_R \gg m_{\rm weak}$, here the first non-vanishing splitting of the two pairs of sneutrino eigenvalues is linear in the suppression parameter $(M_R/m_{\rm weak})$.

In the second scenario discussed in Sec. II radiative contributions to the Majorana mass for left-handed neutrinos are possible and potentially large. In this scenario, $M_R$ is small, i.e., $M_R \ll m_{\rm weak}$, but $BM_R$, induced by the operator (4), is larger than all other entries in the sneutrino mass matrix if $\mathcal{F}_Z$ is of the order of the maximal VEV inducing supersymmetry breaking. For coefficients $z_\nu$ of $\mathcal{O}(1)$, a value of $\mathcal{F}_Z$ smaller than the typical VEV $\sim m_{\rm weak} M_P$ is required in order to avoid a tachyonic sneutrino state. When $\mathcal{F}_Z$ is minimally reduced, the splittings in the two pairs of eigenvalues of the two matrices $\mathcal{M}^2_\pm$ is of $\lesssim \mathcal{O}(m^2_{\rm weak})$. However, it is easy to convince oneself, at least *a posteriori*, that the requirement $m_{\nu_i} \lesssim 1\,{\rm eV}$ implies $BM_R < m^2_{\rm weak}$. Therefore, the approximated expressions in Eqs. (23), (24), and (25) still hold in this case.

### B. Neutralino mass matrix

The neutralino sector of this model is also more involved than in the minimal supersymmetric extension of the SM. The spontaneous breaking of $U(1)_{B-L}$ is encapsulated in the superpotential terms

$$W = \frac{1}{\sqrt{2}}\, Y \left(\Phi\bar{\Phi} - v^2_\Phi\right). \tag{26}$$

Here, $Y$ is a chiral superfield neutral under $U(1)_{B-L}$ that forces $\Phi$ and $\bar{\Phi}$ to acquire VEV's $\langle\Phi\rangle = \langle\bar{\Phi}\rangle = v_\Phi$, which can be assumed real and positive. The two fields $\Phi$ and $\bar{\Phi}$ can be reexpressed in terms of a Goldstone chiral multiplet $\Psi$, which is absorbed by the $U(1)_{B-L}$ gauge multiplet $X$, and a chiral field $K$. The effect of this super-Higgs mechanism is that the gauge multiplet acquires mass

$$M_G = 2g_X|X_\Phi|v_\Phi\,, \tag{27}$$

where $g_X$ is the $U(1)_{B-L}$ gauge coupling and $X_\Phi$ the $U(1)_{B-L}$ charge of the field $\Phi$. The superpotential in (26) reduces to

$$W = \frac{1}{2\sqrt{2}}YK^2 + v_\Phi YK\,. \tag{28}$$

(Notice that a potentially dangerous coupling $YH\bar{H}$, which could shift the scale of the SM breaking up to $M_G$, can be easily forbidden by making use of an $R$ symmetry.)

The implementation of a spontaneous breaking of $U(1)_{B-L}$ requires therefore the inclusion of at least three SM neutral singlet superfields. As a consequence, new neutralino states are present in this model in addition to the usual $\tilde{B}$, $\tilde{W}_3$, $\tilde{H}_0$, and $\tilde{\bar{H}}_0$. For the three new singlet superfields $\Phi$, $\bar{\Phi}$, and $Y$, the new neutralino states are $\tilde{\Psi}$, $\tilde{K}$, $\tilde{Y}$, and the $U(1)_{B-L}$ gaugino $\tilde{X}$. On the basis $\left(\tilde{B}\,\tilde{X}\,\tilde{\Psi}|\tilde{K}\,\tilde{Y}\,||\,\tilde{W}_3\,\tilde{H}_0\,\tilde{\bar{H}}_0\right)^T$, the neutralino mass matrix, when normalized as

$$-\mathcal{L}_{\rm mass} = 1/2\left(\tilde{B}\,\tilde{X}\,\tilde{\Psi}|\tilde{K}\,\tilde{Y}\,||\,\tilde{W}_3\,\tilde{H}_0\,\tilde{\bar{H}}_0\right)\mathcal{M}_{\rm neutr}\left(\tilde{B}\,\tilde{X}\,\tilde{\Psi}|\tilde{K}\,\tilde{Y}\,||\,\tilde{W}_3\,\tilde{H}_0\,\tilde{\bar{H}}_0\right)^T\,, \tag{29}$$

acquires the form

$$\mathcal{M}_{\rm neutr} = \left(\begin{array}{ccc|cc||ccc} m_{\widetilde{B}} & m_{\rm mix} & 0 & 0 & 0 & 0 & gv & gv \\ m_{\rm mix} & m_{\widetilde{X}} & M_G & 0 & 0 & 0 & gv & gv \\ 0 & M_G & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & v_\Phi & 0 & 0 & 0 \\ 0 & 0 & 0 & v_\Phi & 0 & 0 & 0 & 0 \\ \hline\hline 0 & 0 & 0 & 0 & 0 & m_{\widetilde{W}} & gv & gv \\ gv & gv & 0 & 0 & 0 & gv & 0 & \mu \\ gv & gv & 0 & 0 & 0 & gv & \mu & 0 \end{array}\right). \tag{30}$$

In Eq. (30), the two-component notation for the spinor fields is used. The entry $M_G$ is the Dirac-type mass that couples $\tilde{X}$ to $\tilde{\Psi}$, whereas the Majorana-type mass $m_{\widetilde{X}}$ for the gaugino $\tilde{X}$ is a soft supersymmetry-breaking parameter of the order of the weak scale. The entry $m_{\rm mix}$, also at the electroweak scale, is a mass terms that mixes the U(1)$_Y$ and U(1)$_{B-L}$ gauginos. The generic symbol $gv$ stays here for the product of a SU(2)$\times$ U(1) gauge coupling times one of the two SM VEV's and some normalization factor, and it is not always the same in the different entries of $\mathcal{M}_{\rm neutr}$. The diagonalization of this matrix through a unitary transformation ($D^\dagger \mathcal{M}_{\rm neutr} D$) leads to eight mass eigenstates $\tilde{\chi}^0_i$, related to the current eigenstates through

$$\left( \tilde{B} \;\; \tilde{X} \;\; \tilde{\Psi} \middle| \tilde{K} \;\; \tilde{Y} \middle\| \tilde{W}_3 \;\; \tilde{H}_0 \;\; \tilde{\bar{H}}_0 \right)^T = D \left( \tilde{\chi}^0_1 \;\; \tilde{\chi}^0_2 \;\; \tilde{\chi}^0_3 \;\; \tilde{\chi}^0_4 \;\; \tilde{\chi}^0_5 \;\; \tilde{\chi}^0_6 \;\; \tilde{\chi}^0_7 \;\; \tilde{\chi}^0_8 \right)^T . \tag{31}$$

Of these mass eigenstates, two are degenerate with mass $\pm v_\Phi$, and two are almost degenerate, at the scale $M_G$. Their masses differ by an overall sign; the splitting in the absolute value of these masses is of order $m_{\rm weak}$. The remaining four eigenstates are at the electroweak scale.

In order to visualize the relevance of these states for the radiative generation of neutrino masses, in particular of Dirac type, it is convenient to take the limit $v \to 0$ in the neutralino mass matrix. Mixing terms due to SU(2) Higgsino-gaugino couplings as well as SU(2)-gaugino-U(1)-gaugino couplings, which are of order $(m_Z/\tilde{m})^2$, with $\tilde{m}$ a soft supersymmetry breaking parameter, are in this case neglected. In this limit, $\mathcal{M}_{\rm neutr}$ reduces to a block-diagonal matrix. The lower $3\times 3$ block has no relevance for the radiative generation of neutrino masses, since no $SU(2)$ Higgsino-neutrino-sneutrino couplings are possible in the absence of tree-level neutrino Yukawa couplings. The central $2\times 2$ block turns out to give a vanishing contribution to neutrino masses. Therefore, in this limit, the $3\times 3$ upper block is the only one of interest to induce Dirac neutrino mass operators. Fermionic and scalar components of the singlets $\bar{N}$, indeed, couple only to the gaugino $\tilde{X}$ and the field $\tilde{K}$. However, once the rotation (31) is performed, all mass eigenstates $\tilde{\chi}^0_i$ participate in the vertex $\bar{\nu}_R \tilde{\chi}^0_i \tilde{\nu}_R$, with couplings that are larger for the heavy $\tilde{\chi}^0_i$ states. Two of these $\tilde{\chi}^0_i$ states are a linear combination of $\tilde{K}$ and $\tilde{Y}$ only, obtained through a rotation at $45^0$ of $\tilde{K}$ and $\tilde{Y}$. These states have exactly opposite masses $\pm v_\Phi$ and their contributions to Dirac mass operators cancel identically. A similar feature holds for the contribution coming from the two states that are mainly $\tilde{X}$ and $\tilde{\Psi}$. In this case, however, the cancellation is not exact, but it is spoiled by powers of the suppression factor $(m_{\rm weak}/M_G)$, where $m_{\rm weak}$ can be any of the remaining parameters in the upper $3\times 3$ block of $\mathcal{M}_{\rm neutr}$, i.e. $m_{\widetilde{B}}$, $m_{\rm mix}$, or $m_{\widetilde{X}}$. It turns out that the lowest order suppression factor comes with power *2*. As will be stressed later, this fact has important phenomenological consequences.

We close this section by giving explicitly eigenvalues and eigenvectors of the $3\times 3$ upper block in the neutralino mass matrix (30), assuming that its elements are real. The three eigenvalues are

$$m_{\tilde{\chi}^0_0} = m_{\widetilde{B}}\,, \qquad m_{\tilde{\chi}^0_\pm} = M_G \left\{ \pm 1 + \alpha \frac{x}{2} \pm \alpha^2 \left( \frac{x^2}{8} + \frac{z^2}{2} \right) \right\} , \tag{32}$$

where $\alpha$, $z$, and $x$ are

$$\alpha = \frac{m_{\widetilde{B}}}{M_G}\,, \qquad z = \frac{m_{\rm mix}}{m_{\widetilde{B}}}\,, \qquad x = \frac{m_{\widetilde{X}}}{m_{\widetilde{B}}}\,. \tag{33}$$

At the same level of precision in the small parameter $\alpha$, the reduced $3\times 3$ diagonalization matrix, defined by

$$\left(\,\tilde{B}\;\;\tilde{X}\;\;\tilde{\Psi}\,\right)^T = D_{(3)}\left(\,\tilde{\chi}^0_0\;\;\tilde{\chi}^0_+\;\;\tilde{\chi}^0_-\,\right)^T , \tag{34}$$

is

$$D_{(3)} = \begin{pmatrix} 1-\alpha^2\frac{z^2}{2} & \frac{z}{\sqrt{2}}\left[\alpha+\alpha^2\left(1-\frac{x}{4}\right)\right] & \frac{z}{\sqrt{2}}\left[\alpha-\alpha^2\left(1-\frac{x}{4}\right)\right] \\ -\alpha^2 z & \frac{1}{\sqrt{2}}\left[1+\alpha\frac{x}{4}-\alpha^2\frac{x^2}{32}\right] & \frac{1}{\sqrt{2}}\left[-1+\alpha\frac{x}{4}+\alpha^2\frac{x^2}{32}\right] \\ -\alpha z & \frac{1}{\sqrt{2}}\left[1-\alpha\frac{x}{4}-\alpha^2\left(\frac{x^2}{32}+\frac{z^2}{2}\right)\right] & \frac{1}{\sqrt{2}}\left[1+\alpha\frac{x}{4}-\alpha^2\left(\frac{x^2}{32}+\frac{z^2}{2}\right)\right] \end{pmatrix} . \tag{35}$$

## V. EFFECTIVE DIRAC AND MAJORANA NEUTRINO MASSES

We are now in a position to evaluate Dirac and Majorana mass couplings $m_D$ and $m_L$. Since Majorana masses violate lepton number, they can only be generated for nonvanishing $BM_R$. In contrast, Dirac masses can be obtained also for $M_R = 0$. In the following, we give expressions for $m_D$ and $m_L$ classified according to the value of $M_R$. As in the previous section, also here the flavor changing structure of the soft parameters in the sneutrino potential is ignored. All the following expressions for $m_D$ and $m_L$ hold for each generation. A generalization to the case in which $A$, $B$, $m_{\tilde{l}}$, and $m_{\tilde{\nu}}$, have a nontrivial matrix structure is technically straightforward but leads to more involved formulas.

### A. Dirac neutrino mass $m_D$, $M_R \simeq 0$

The radiatively generated Dirac mass $m_D$ is evaluated in the mass eigenbasis for neutralinos and sneutrinos. The corresponding diagram is shown in Fig. 1, where the sneutrino soft parameter $A$, which breaks chiral symmetries, and the neutralino mass $m_{\tilde{\chi}^0_j}$, which connects left- and right-chiral fermions, are shown as mass insertions.

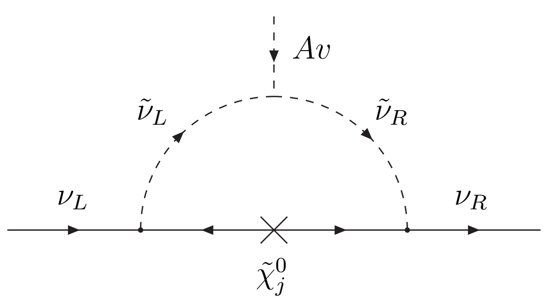


FIG. 1. Diagram contributing to the Dirac neutrino mass $m_D$.

The Majorana mass $M_R$, which couples the singlets $\bar{N}$, is assumed to exist at the tree level, but to be small. Therefore, the approximation in Eq. (20) is sufficient for the determination of the sneutrino mass eigenstates. They are, in this case, only two, $\tilde{\nu}_1$ and $\tilde{\nu}_2$, and are both at the electroweak scale. The current eigenstate vertex $\bar{\nu}_R \tilde{X} \tilde{\nu}_R$ has a coupling $-i\sqrt{2} g_X X_{\bar{N}}$ where $X_{\bar{N}}$ is the charge of the singlet superfield $\bar{N}$ under $U(1)_{B-L}$. The gaugino $\tilde{X}$ couples also to the left-handed neutrinos and sneutrinos with a similar coupling $-i\sqrt{2} g_X X_L$. A straightforward calculation of the Dirac neutrino mass, in the one-generation case, then, yields

$$m_D = \frac{(Av)}{8\pi^2} \sum_{j=1}^{6} (g_X X_{\bar{N}} D_{2j})\, m_{\tilde{\chi}_j^0} \left\{ g_Y Y_L D_{1j} + g_X X_L D_{2j} + g_2 T_{3L} D_{6j} \right\} I(m_{\tilde{\nu}_1}^2, m_{\tilde{\nu}_2}^2, m_{\tilde{\chi}_j^0}^2)\,, \quad (36)$$

where the function $I(m_1^2, m_2^2, m_3^2)$ is defined in [33], $Y_L$ is the hypercharge of the SU(2) leptonic doublet, $T_{3L}$ is the isospin of the neutral component of this doublet, and $D$ is the neutralino diagonalization matrix defined in Eq. (31).

This result becomes particularly transparent when the limit $v \to 0$ is taken in the neutralino mass matrix. In this limit, the mixed contribution of $\tilde{W}_3$ and heavy neutralinos vanishes, and the third term in the curly bracket of Eq. (36) drops out. Thus, in the approximation of vanishing phases in the neutralino mass matrix, the Dirac neutrino mass becomes

$$\begin{aligned} m_D \simeq\ & \frac{g_Y g_X}{8\pi^2} Y_L X_{\bar{N}}\, (Av)\, m_{\rm mix} \left\{ -\left(\frac{m_{\widetilde{B}}}{M_G}\right)^2 I(m_{\tilde{\nu}_1}^2, m_{\tilde{\nu}_2}^2, m_{\widetilde{B}}^2) + I(m_{\tilde{\nu}_1}^2, m_{\tilde{\nu}_2}^2, M_G^2) \right\} \\ & + \frac{g_X^2}{8\pi^2} X_L X_{\bar{N}} \left(\frac{Av}{m_{\widetilde{B}}}\right) \left(\frac{m_{\widetilde{X}}}{m_{\widetilde{B}}}\right) \left(\frac{m_{\widetilde{B}}}{M_G}\right)^2 , \end{aligned} \quad (37)$$

where higher orders in the expansion parameter $(m_{\widetilde{B}}/M_G)$ have been neglected. Notice that in the absence of a mass term mixing the $U(1)_Y$ and $U(1)_{B-L}$ gauginos, a contribution to the Dirac mass $m_D$ comes from pure $\tilde{X}$-$\tilde{X}$ exchange. Finally, in the approximation $m_{\tilde{\nu}_1}^2 \simeq m_{\tilde{\nu}_2}^2 \simeq m_{\widetilde{B}}^2 \ll M_G^2$, the above result can be cast in the simple form

$$m_D \sim \frac{g_X}{8\pi^2} X_{\bar{N}} \left(\frac{Av}{m_{\widetilde{B}}}\right) \left(\frac{m_{\widetilde{B}}}{M_G}\right)^2 \left\{ g_Y Y_L \left(\frac{m_{\rm mix}}{m_{\widetilde{B}}}\right) \left[ 2\log\left(\frac{M_G}{m_{\widetilde{B}}}\right) - \frac{3}{2} \right] + g_X X_L \left(\frac{m_{\widetilde{X}}}{m_{\widetilde{B}}}\right) \right\}. \quad (38)$$

For soft supersymmetry-breaking terms at the electroweak scale, the Dirac neutrino mass $m_D$, besides being suppressed by a loop factor, is moved away from $m_{\rm weak}$ by the ratio $(m_{\rm weak}/M_G)^2$. The behavior $m_D \simeq (m_{\rm weak}/8\pi^2)(m_{\rm weak}/M_G)^2$ is quite general and it is not limited to the choice of gauge group $U(1)_{B-L}$ made here. It has to be observed, however, that the suppression factor $(m_{\rm weak}/M_G)^2$ is actually $(\tilde{m}/M_G)^2$, where $\tilde{m}$ is a generic soft supersymmetry-breaking parameter. Therefore, because of its dependence on a fixed scale $M_G$, $m_D$ tends to grow in the superpartner decoupling limit. The effectiveness of the suppression factor $(\tilde{m}/M_G)^2$ is recovered only when also $M_G$ is increased and/or the trilinear parameters $A$ are decreased.

### B. Dirac neutrino mass $m_D$, $M_R \gg m_{\rm weak}$

If the Majorana mass $M_R$ is non-negligible, then $CP$-even and $CP$-odd components of the sneutrino states are split by lepton flavor-violating mass terms. Therefore, when calculating

$m_D$, the general form in Eq. (15) for the sneutrino mass matrix has to be used, or at least that in Eq. (16) obtained in the basis in which the $A$ and $B$ parameters are real. In this basis, $m_D$ gets the form

$$m_D = \frac{(Av)}{16\pi^2}\sum_{j=1}^{6}(g_X X_{\bar N} D_{2j})\, m_{\tilde\chi^0_j}\,\{g_Y Y_L D_{1j} + g_X X_L D_{2j} + g_2 T_{3L} D_{6j}\} \times\left[I(m^2_{\tilde\nu_{+,1}}, m^2_{\tilde\nu_{+,2}}, m^2_{\tilde\chi^0_j}) + I(m^2_{\tilde\nu_{-,1}}, m^2_{\tilde\nu_{-,2}}, m^2_{\tilde\chi^0_j})\right]. \qquad (39)$$

When taking the limit $v \to 0$ in the neutralino mass matrix, also in this case, the mixed contribution of $\widetilde W_3$ and heavy neutralinos vanishes. Moreover, if $M_R \simeq M_G$, much above the electroweak scale, while $m_{\tilde\nu_{-,1}} \simeq m_{\tilde\nu_{+,1}} \simeq m_{\tilde l} \simeq m_{\widetilde B}$, the above expression reduces to

$$m_D \sim \frac{g_X}{8\pi^2} X_{\bar N}\left(\frac{Av}{m_{\widetilde B}}\right)\left(\frac{m_{\widetilde B}}{M_R}\right)^2 \times\left\{g_Y Y_L\left(\frac{m_{\rm mix}}{m_{\widetilde B}}\right)\left[2\log\left(\frac{M_R}{M_G}\right)\right] + g_X X_L\left(\frac{m_{\widetilde X}}{m_{\widetilde B}}\right)\left[2\log\left(\frac{M_R}{M_G}\right) - 1\right]\right\}, \qquad (40)$$

in the approximation of vanishing phases in the neutralino mass matrix. Although different in some details, this expression for $m_D$ is qualitatively very similar to that obtained when $M_R \simeq 0$. The double suppression $(m_{\rm weak}/M_G)^2$ seems, therefore, a typical feature of radiatively induced Dirac masses. It holds, indeed, for any value of $M_R$.

### C. Majorana neutrino mass $m_L$, $M_R \gg m_{\rm weak}$

The Majorana neutrino mass $m_L$ violates lepton number and must have a direct dependence on the soft parameter $B$, which is the only lepton-number violating parameter in the sneutrino potential. [5] The diagram needed to obtain $m_L$ is shown in Fig. 2. As in the diagram of Fig. 1, the parameters $A$ and $B$ as well as the relevant gaugino mass are shown as mass insertions, although the calculation is done using the mass eigenstate formalism. The neutrino mass is directly proportional to the splitting of each of the two pairs of eigenvalues of the sneutrino mass matrix (16), induced by $B$. The largest contribution to the Majorana mass comes from the splitting in the lightest eigenvalues. The splitting in the two heavy eigenvalues, which appears at a lower order in the expansion parameter $m_{\tilde l}/M_R$ than the splitting in the two light eigenvalues, gives a much more suppressed contribution. The resulting expression is

[5] A different possibility was considered in the last reference in [33]. In the absence of additional singlets $\bar N$, a lepton-number violating soft supersymmetry-breaking trilinear term was allowed. A mass splitting among left-handed sneutrinos was induced at the one-loop level and a Majorana neutrino mass $m_L$ at the two-loop level.

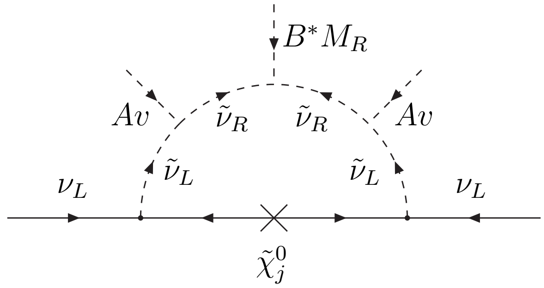


FIG. 2. Diagram contributing to the Majorana neutrino mass $m_L$.

$$m_L = \frac{1}{4\pi^2} \sum_{j=1}^{6} (g_Y Y_L D_{1j} + g_X X_L D_{2j} + g_2 T_{3L} D_{6j})\, m_{\tilde{\chi}_j^0}\, (g_Y Y_L D_{1j} + g_X X_L D_{2j} + g_2 T_{3L} D_{6j}) \times \left(\frac{Av}{m_{\tilde{l}}}\right)^2 \left(\frac{B}{m_{\tilde{l}}}\right) \left(\frac{m_{\tilde{l}}}{M_R}\right)^3 I(m^2_{\tilde{\nu}_{+,1}}, m^2_{\tilde{\nu}_{-,1}}, m^2_{\tilde{\chi}_j^0}) \,. \tag{41}$$

When taking the limit $v \to 0$ in the neutralino mass matrix, the contribution from mixed propagators $\tilde{B}$-$\tilde{W}_3$ drops out and only the pure $\tilde{B}$-$\tilde{B}$, $\tilde{X}$-$\tilde{X}$, and $\tilde{W}_3$-$\tilde{W}_3$ propagators survive. Furthermore, assuming a real neutralino mass matrix, in the limit $m^2_{\tilde{\nu}_{+,1}} \simeq m^2_{\tilde{\nu}_{-,1}} \simeq m^2_{\tilde{l}} \simeq m^2_{\widetilde{B}} \simeq m^2_{\widetilde{W}}$, as well as $M_G \simeq M_R$ for the functions $I(.,.,.)$, the previous expression becomes

$$m_L \simeq \frac{1}{8\pi^2} \left(\frac{Av}{m^2_{\tilde{l}}}\right)^2 \left(\frac{B}{m_{\tilde{l}}}\right) \left(\frac{m_{\tilde{l}}}{M_R}\right)^3 \left\{ (g_Y Y_L)^2\, m_{\widetilde{B}} + (g_2 T_{3L})^2\, m_{\widetilde{W}} \right\} \,. \tag{42}$$

It is interesting to see that the suppression $(m_{\rm weak}/M_R)$ comes with a higher power than in the expression for the Dirac mass $m_D$. It is, indeed, $m_L \sim (m_{\rm weak}/8\pi^2)(m_{\rm weak}\, \tilde{m}^2/M_R^3)$. Also in this case, $m_L$ tends to increase in the superpartner decoupling limit and its smallness can be retrieved by tuning the parameters $A$ and $B$ to be very small or raising the value of $M_R$.

### D. Majorana neutrino mass $m_L$, $M_R \ll m_{\rm weak}$

If $M_R$ is much smaller than the electroweak scale, the Majorana neutrino mass $m_L$ is not suppressed by the large scale $\sim M_G$. In this case, all particles exchanged in the loop in Fig. 2 have masses of the order of the electroweak scale, and they all contribute to $m_L$. Thus, the suppression factor only comes from $(M_R/m_{\rm weak})$ when all the supersymmetry-breaking parameters are around the weak scale. The Majorana neutrino mass is given by

$$m_L = \frac{1}{4\pi^2} \sum_{j=1}^{6} (g_Y Y_L D_{1j} + g_X X_L D_{2j} + g_2 T_{3L} D_{6j})\, m_{\tilde{\chi}_j^0}\, (g_Y Y_L D_{1j} + g_X X_L D_{2j} + g_2 T_{3L} D_{6j}) \times (B M_R) \frac{(Av)^2}{(m^2_{\tilde{\nu}_2} - m^2_{\tilde{\nu}_1})^2} \left[ I(m^2_{\tilde{\nu}_1}, m^2_{\tilde{\nu}_1}, m^2_{\tilde{\chi}_j^0}) + I(m^2_{\tilde{\nu}_2}, m^2_{\tilde{\nu}_2}, m^2_{\tilde{\chi}_j^0}) - 2 I(m^2_{\tilde{\nu}_1}, m^2_{\tilde{\nu}_2}, m^2_{\tilde{\chi}_j^0}) \right] \,, \tag{43}$$

where $m^2_{\tilde{\nu}_{1,2}}$ are defined in Eq. (21). When taking the limit $v \to 0$ in the neutralino mass matrix, the contribution from mixed propagators $\tilde{B}$-$\tilde{W}_3$ drops out and the dominant contributions come from the $\tilde{B}$-$\tilde{B}$ and $\tilde{W}_3$-$\tilde{W}_3$ propagators. It is easy to see from Eq. (43) that the Majorana neutrino mass $m_L$ generated when $M_R \ll m_{\rm weak}$ is of order $(1/8\pi^2)(BM_R/m_{\rm weak})$.

The expression (43) is valid also for the radiative contribution to $m_L$ obtained in the second (tree-level) scenario of Sec. II, in which no $U(1)_{B-L}$ symmetry is imposed, and generically for the contributions obtained when $M_R \ll m_{\rm weak}$ and $BM_R \lesssim m^2_{\rm weak}$. As was already argued in Sec. IV A, acceptable values of $m_L$ arise, in general, when $BM_R$ is not too close to $m^2_{\rm weak}$. Although the supersymmetry-breaking parameter $B$ is very large, the approximations made in Sec. IV A, as well as the resulting expression (43), are still applicable.

## VI. EFFECTIVE YUKAWA NEUTRINO AND SNEUTRINO INTERACTIONS

Yukawa interaction terms Higgs boson–lepton doublet–right-handed neutrino are also induced radiatively. The diagrams for the neutral Higgs boson–left-handed-neutrino–right-handed-neutrino interaction term are similar to that in Fig. 1, with the external scalar line being that of a physical neutral Higgs boson. Analogous diagrams generate also the vertex of charged Higgs bosons–charged leptons–right handed neutrinos. The interaction terms $\nu_L\nu_L H^0 H^0$ as well as $eeH^+H^+$ are obtained from variations of the diagram in Fig. 2. The main characteristic of the couplings associated with these terms is that they are momentum dependent with, in general, complicated form factors. Although different from $m_D/v$ and $m_L/v^2$, they have, however, the dimensionality and order of magnitude of these ratios.

For a Dirac mass in the eV range, for example, the effective Yukawa coupling is $\sim 10^{-11}$. The smallness of this coupling is now *natural* and, as the smallness of $m_D$, owes its origin to the seesaw pattern of widely different scales for the neutralinos entering in the loop calculation. In the SM, when the Higgs mechanism is the only way to obtain small neutrino masses, such a small value can only be imposed by hand. In the traditional seesaw mechanism, the couplings for Yukawa interaction terms are of $\mathcal{O}(1)$. While the phenomenological relevance of these terms for possible collider signals is marred by the presence of right-handed fields at scales $\sim 10^{12}$ GeV, these large couplings have subtle consequences in other sectors. For example, they alter the pattern of Yukawa couplings unification [38]. Moreover, they lift the predictions for lepton-flavor violating processes involving charged leptons, such as the decay $\mu \to e\gamma$, by inducing sizable splittings among charged sleptons of different generations. Indeed, these processes, which are realized at the quantum level, may be detectable if the mass of the charged sleptons exchanged in the loops is not too heavy [39].

In the scenarios presented here, with intrinsically small effective Yukawa couplings, large rates for lepton-flavor-violating processes are obtained if a large splitting among the charged-slepton masses of different generations exists already at the Planck scale or at the scale of $U(1)_{B-L}$ breaking. Notice that, although small, these effective Yukawa couplings, obtained radiatively, are, in general, much larger than those obtained at the tree level from the operators in Eq. (2). Indeed, these tree-level Yukawa couplings are naturally $\sim 10^{-13}$; see the discussion in Secs. II and VIII. In some specific cases, however, they may be larger and even comparable to those obtained radiatively. In this case, the actual couplings of neutrino Yukawa interactions are of mixed origin. They remain, however, very small.

The dimensionful coupling $m_L/v^2$ for the interaction terms $\nu_L\nu_L H^0 H^0$ and $eeH^+H^+$ is $\sim (10^{13}\,\mathrm{GeV})^{-1}$ for $m_L \sim 1\,\mathrm{eV}$. This is the same dimensionful suppression factor that can be obtained by integrating out right-handed neutrinos of mass $M_R \sim 10^{13}\,\mathrm{GeV}$ if these couple at the tree level to Higgs and lepton doublets with couplings of $\mathcal{O}(1)$. Couplings that may arise from operators such as those in Eq. (11), in the case of a gauged $U(1)_{B-L}$ symmetry or from operators such as the first in Eq. (1) when $U(1)_{B-L}$ is not introduced, are much smaller than the effective couplings obtained radiatively.

Similar considerations hold also for Higgsino-sneutrino-neutrino operators, which also arise at the quantum level [33]. Their couplings are, in general, so small as to be irrelevant for phenomenology.

Large are, on the contrary, the trilinear sneutrino soft terms $A$, which can be as large as the electroweak scale. In this case, relatively large interaction terms $AH^0\,\tilde{\nu}_R^*\,\tilde{\nu}_L$ and $AH^+\,\tilde{\nu}_R^*\,\tilde{e}_L$ are present. They may possibly affect searches for neutral and charged Higgs bosons. Particularly interesting is, in this respect, the situation in which the right-handed sneutrino $\tilde{\nu}_R$ is very light. (For the effect of light sneutrinos in searches of the charged Higgs boson, within a generic supersymmetric model, see [40].) A dedicated study of this possibility is certainly important and is left for future work.

## VII. PHYSICAL NEUTRINO MASSES

It was shown in the previous sections that small neutrino masses can be naturally obtained in supersymmetric models with right-handed neutrinos, in which (i) the lowest order neutrino Yukawa operators are forbidden; (ii) effective Yukawa couplings are induced through higher-dimensional operators by a spurion field $Z$ with a supersymmetry-conserving VEV much smaller than the supersymmetry-violating one, and (iii) a $U(1)_{B-L}$ gauge symmetry may be introduced. In this class of models, contributions to Dirac and Majorana neutrino masses $m_D$ and $m_L$ entering the neutrino mass matrix

$$\begin{pmatrix} m_L & m_D \\ m_D & M_R \end{pmatrix} \tag{44}$$

are generated at the tree level and/or at the quantum level. The overall results obtained for neutrino masses can be classified according to the value of the right-handed neutrino mass $M_R$.

(1) If $M_R$ is large enough to be hierarchically split from $m_{\mathrm{weak}}$, $M_R \gg m_{\mathrm{weak}}$, the complete or largest contributions to $m_L$ and $m_D$ are, naturally, of radiative origin if the light neutrino states are assumed to be $\sim 1\,\mathrm{eV}$. This case is naturally realized when a $U(1)_{B-L}$ gauge symmetry broken at a large scale $M_G$ is introduced. If $M_R \simeq M_G$, the Majorana mass $m_L$, with its dependence $m_L \sim (m_{\mathrm{weak}}/8\pi^2)(m_{\mathrm{weak}}/M_R)^3$, is much smaller than the Dirac mass $m_D$, which is $\sim (m_{\mathrm{weak}}/8\pi^2)(m_{\mathrm{weak}}/M_G)^2$. Nevertheless, the eigenstates $\nu_i$ have mass dominated by $m_L$. The eigenstates $n_i$ are, as in the usual seesaw mechanism, at the large

scale $M_R$. The six eigenstates are Majorana fermions. The mixing angle $\theta_\nu$ between light and heavy states, is $|\sin 2\theta_\nu| \sim 2|m_D|/M_R$. Therefore, the states $\nu_i$ and $n_i$ are, respectively almost pure active and sterile neutrinos. Light states $\nu_i$ at the eV scale require $M_R(\sim M_G) \sim 100\,\mathrm{TeV}$. This scale, however, can be decreased if the soft parameters $A$ and $B$ are suppressed with respect to $m_{\rm weak}$. Notice that for light neutrinos in the sub-eV range, the tree-level contribution to $m_D$ may be non-negligible with respect to the radiative one and affect the mixing angle.

(2) If $M_R$ is around the electroweak scale, $m_L$ is, in general, suppressed with respect to $m_{\rm weak}$ only by numerical loop factors. A drastic tuning of $A$ and/or $B$ is then required to reduce $m_L$ to the eV scale. [Small values of these parameters may naturally occur in a gauge-mediated supersymmetry-breaking scenario (see Sec. VIII).] Also this case, as the previous one of large $M_R$, can be realized when a $U(1)_{B-L}$ gauge symmetry is introduced.

(3) If $M_R$ is much smaller than the electroweak scale, the radiative contribution to $m_L$ is given by $\sim (1/8\pi^2)(BM_R/m_{\rm weak})$. The Dirac mass $m_D$ has a tree-level contribution $\sim \mathcal{A}_Z v/M_P$ with $v$ an electroweak VEV. In addition, if no $U(1)_{B-L}$ symmetry is introduced, the left- and right-handed Majorana masses $m_L \sim v^2/M_P$ and $M_R \sim \mathcal{A}_Z^2/M_P$ are also generated at the tree level as discussed in the second scenario of Sec. II. If a $U(1)_{B-L}$ gauge symmetry is introduced, on the other hand, we have a radiative contribution to the Dirac mass of $\sim (m_{\rm weak}/8\pi^2)(m_{\rm weak}/M_G)^2$. The relative size of $M_R$ with respect to that of $m_D$ has actually rather important physical consequences. Therefore, we further distinguish the following two cases.

(3a) For $M_R \gtrsim m_D$, the six Majorana eigenstates $(\nu_i, n_i)$ have masses $\sim m_L + m_D^2/M_R$, $M_R$, if there is a hierarchy between $M_R$ and $m_D$, or $(1/2)\{M_R \pm \sqrt{M_R^2 - 4m_D^2}\}$, if $M_R$ is comparable to $m_D$. The first case can be realized through the tree-level or the radiative mechanism, or both; the second is typical of the radiative generation of neutrino masses with a $U(1)_{B-L}$ gauge symmetry. The mixing angle between $\nu_i$ and $n_i$ ranges from small, for $M_R > m_D$, to maximal, for $M_R \sim m_D$. Notice that $M_R \sim 10^{-3}\,\mathrm{eV}$ and $m_D \sim 10^{-5}\,\mathrm{eV}$ could explain the solar neutrino problem with an oscillation $\nu_e \to \nu_s$ and a small mixing angle. In order to avoid the same pattern of oscillation also in the other two generations, a particular flavor structure of the trilinear $A$ terms and/or of the different $M_R$ has to be implemented.

(3b) The case $M_R \ll m_D$ is typically realized in the radiative mechanism of neutrino Dirac mass generation. In this case, almost degenerate Majorana states $(\nu_i, n_i)$ are obtained with a nearly maximal mixing. Their masses are of order $m_D \sim (m_{\rm weak}/8\pi^2)(m_{\rm weak}/M_G)^2$, so that a scale of $B-L$ violation of $\sim 10^7\,\mathrm{GeV}$ is required to obtain neutrino masses in the eV range.

(4) Finally, there is the possibility $M_R = 0$, for which the Majorana states $(\nu_i, n_i)$ have masses $\pm m_D$ and combine into a Dirac neutrino, for each family. As discussed in the previous sections, in both mechanisms of neutrino-mass generation, this possibility can be realized by imposing an additional symmetry, such as a lepton-number symmetry or its discrete subgroup given in Eq. (3). Notice that purely Dirac neutrinos emerge in this case, even when a $U(1)_{B-L}$ gauge symmetry is imposed. From the phenomenological point of view this case is very different from all the others discussed above. The observed oscillation structure

of atmospheric and solar oscillations can then be explained only as a flavor oscillation. It is obviously very difficult to accommodate in such a scenario the results from the LSND experiment. Naturally, no signal is expected from the $\beta\beta$ decay. Moreover, relatively large neutrino dipole moments are predicted, i.e., dipole moments which are not loop suppressed with respect to neutrino masses (see the discussion on quark and lepton dipole moments in a scenario of radiative generation of fermion masses [33]).

Both mechanisms for generating small neutrino masses proposed in this paper offer a very fertile ground in which widely different scenarios of neutrino masses and mixing can be implemented. If the lowest order tree-level Yukawa couplings exist, then the loops calculated above provide corrections to the tree-level neutrino masses. For unsuppressed Yukawa couplings, i.e., for couplings of $\mathcal{O}(1)$, these corrections are negligibly small once the scale $M_R$ is raised, as in the usual seesaw case, to get tree-level masses of the correct order of magnitude. If tree-level Yukawa couplings are, however, suppressed by some other mechanism, as explained in Secs. II and VIII, then a large variety of neutrino spectra is possible. The fact that the lowest-order neutrino Yukawa operators are vanishing is therefore a key ingredient to unravel a much wider range of possibilities than that offered by the traditional seesaw mechanism.

## VIII. MODEL EMBEDDING

In this section, an explicit model that generates the desired hierarchy between the supersymmetry-conserving and the supersymmetry-violating VEV of the $Z$ field, i.e., $\mathcal{A}_Z \ll \mathcal{F}_Z$, is presented. We recall that $\mathcal{A}_Z$ is responsible for the tree-level generation of neutrino masses, whereas $\mathcal{F}_Z$ gives rise to the sneutrino soft trilinear $A$ parameter. This, together with a neutralino mass, induces quantum contributions to the Dirac mass $m_D$. The same parameter $A$, together with the soft bilinear $B$ parameter and a neutralino mass, induces a radiative contribution to the Majorana mass $m_L$. Comments on the size of the parameters $A$ and $B$ in various supersymmetry-breaking scenarios are also made.

The model is based on a supersymmetric SU(2) gauge theory with four doublet chiral superfields $Q_i$ and six singlet superfields $Z^{ij} = -Z^{ji}$ where $i, j$ are flavor indices ($i, j = 1, \cdots, 4$) [41]. All the $Q_i$ and $Z^{ij}$ fields are singlets under the SM gauge groups. Without a superpotential, this theory has a flavor SU$(4)_F$ symmetry. This is explicitly broken to a global SP$(4)_F$ subgroup if the tree-level superpotential [42]

$$W_{\text{tree}} = \lambda Z(QQ) + \lambda_Z Z^a (QQ)_a \tag{45}$$

is introduced. Here, $(QQ)$ and $Z$ are singlets of $SP(4)_F$, while $(QQ)_a$ and $Z^a$ are five-dimensional representations of $SP(4)_F$ ($a = 1, \cdots, 5$). [$Z$ and $Z^a$ are certain linear combinations of the $Z^{ij}$ fields; $(QQ)$ and $(QQ)_a$ combinations of the gauge-invariant operators $(Q_i Q_j)$.] The model has a discrete symmetry $Z_n$ that assigns the following charges to the $Q_i$ and $Z^{ij}$ fields:

$$Z_n\left(Q_i\right) = \frac{1}{2}, \quad Z_n\left(Z^{ij}\right) = -1\,, \tag{46}$$

and that is identified with the symmetry used in Secs. II and IV to forbid tree-level neutrino Yukawa terms $\bar{N}LH$. [6] Then, the $Z$ field can couple to the SM fields through the superpotential in Eq. (2).

The dynamics of the SU(2) gauge theory causes the condensation of the $Q_i$ fields through nonperturbative effects [43]. By integrating out the SU(2) gauge fields together with $Q_i$ and $Z^a$, the low-energy effective superpotential

$$W_{\rm eff} \simeq \lambda \Lambda^2 Z \tag{47}$$

is obtained for $\lambda_Z > \lambda$, where $\Lambda$ is a dynamical scale of the SU(2) gauge interaction. [Notice that there is a mixed $Z_n$–SU(2) anomaly and that $\Lambda$ has therefore a $Z_n$ charge.] Thus, the $Z$ field acquires a nonvanishing $F$ term, $\mathcal{F}_Z \simeq \lambda \Lambda^2 \neq 0$ [41], which induces a sneutrino $A$ parameter of order $\mathcal{F}_Z/M_P$.

The $Z$ direction is flat at the tree level, but it is lifted by loop corrections. The corrections to the Kähler potential from the strong SU(2) gauge interaction are noncalculable and can only be estimated. They give rise to a quadratic term in $Z$ in the scalar potential. If positive, this term, together with a linear term due to supergravity effects, induces a tiny VEV for the scalar component of $Z$ [44]. However, there are also contributions to the mass squared for the $Z$ field near the origin coming from loops of light particles. These are calculable and larger than the noncalculable ones as long as $\lambda$ is in a perturbative regime [45]. The sign of the mass squared is shown to be positive [45], so that $Z = 0$ is a local minimum of the potential in the limit of a global supersymmetry. Thus, the scalar potential for the $Z$ field has the form

$$V \simeq \frac{|\lambda|^4}{16\pi^2} \Lambda^2 |Z|^2 + \left( \lambda \Lambda^2 m_{3/2} Z + {\rm h.c.} \right) , \tag{48}$$

and the nonvanishing VEV for the scalar component of the $Z$ field is

$$\mathcal{A}_Z \simeq \frac{16\pi^2}{\lambda^3} m_{3/2} \, . \tag{49}$$

This VEV, in turn, induces effective tree-level neutrino Yukawa couplings of order $\mathcal{A}_Z/M_P$ through the superpotential term in Eq. (2). If $m_{3/2} \simeq 1\,{\rm TeV}$ and $\lambda = O(1)$, the resulting Dirac neutrino masses, $m_D \simeq 10^{-2}\,{\rm eV}$, are sufficiently large to explain the solar neutrino deficit by the MSW mechanism. If $\lambda$ is somewhat smaller than 1, it is even possible to generate neutrino masses in the range needed by LSND and atmospheric neutrino experiments. Note that this tree-level generation of neutrino masses generically occurs if a chiral superfield $Z$, which couples to SM fields through the superpotential term in Eq. (2), acquires a nonvanishing auxiliary component at the tree level, but has a vanishing scalar component in the global supersymmetry limit.

Thus, the above model can yield the desired size of the $A$ parameter and of the effective tree-level Yukawa couplings through the supersymmetry-violating and supersymmetry-conserving VEV's of the $Z$ field. On the other hand, a $B$ parameter of the order of the

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[6] Here, it is assumed that quadratic (if $n = 2$) and cubic (if $n = 3$) terms in $Z$ are absent.

gravitino mass $m_{3/2}$ is always generated by supergravity effects. In addition, if the $Z_n$ symmetry needed to forbid the renormalizable neutrino Yukawa operators is generic (i.e., $n \neq 2$), the $B$ parameter also has a contribution of order $\mathcal{F}_Z/\mathcal{A}_Z$ from Eq. (9), which is generically much larger than $m_{3/2}$. (In this case, the value of $M_R$ is, however, rather small.) To summarize, in the present model, we obtain the $A$ and $B$ parameters

$$A \simeq \frac{\Lambda^2}{M_P}, \qquad B \simeq \begin{cases} m_{3/2} + \dfrac{\Lambda^2}{16\pi^2 m_{3/2}} & [\text{for } Z_n\,(n \neq 2)], \\ m_{3/2} & [\text{for } Z_2], \end{cases} \tag{50}$$

and the tree-level neutrino Yukawa couplings $y_\nu$:

$$y_\nu \simeq \frac{16\pi^2 m_{3/2}}{M_P}. \tag{51}$$

Here, we have set $\lambda$ and the coupling of the superpotential in Eq. (2) to be of order 1. For simplicity, only the case of $B \simeq m_{3/2}$ will be considered in the following discussion. [7]

It is interesting to compare the size that the above parameters acquire in supergravity-mediated [46] and gauge-mediated [47,48] supersymmetry-breaking scenarios. In the first, the gravitino mass $m_{3/2}$, as well as the $B$ parameter, is at the electroweak scale $\sim 1\,\text{TeV}$. The effective neutrino Yukawa couplings are of order $10^{-13}$ and induce Dirac neutrino masses of order $m_D \simeq 10^{-2}\,\text{eV}$. On the other hand, the $A$ parameter is less constrained. Since the $Z$ field is charged under the $Z_n$ symmetry needed to forbid the lowest order Yukawa operators, its VEV $\mathcal{F}_Z$ cannot give sizable gaugino masses. Therefore, an additional singlet $S$ with a supersymmetry-violating VEV $\mathcal{F}_S \simeq m_{3/2} M_P$ is needed to give soft supersymmetry-breaking mass to all superparticles. Thus, the phenomenological constraint on the supersymmetry-violating VEV of the $Z$ field, $\mathcal{F}_Z$, is only $\mathcal{F}_Z \lesssim \mathcal{F}_S$, and this translates into $A \lesssim m_{3/2}$. Although it is logically possible that the $A$ parameter is much smaller than the electroweak scale, we may naturally expect that $\mathcal{F}_Z$ and $\mathcal{F}_S$ are somehow related to each other. If this is the case, the $A$ parameter is also of order of the gravitino mass. This situation was assumed throughout the previous sections.

In a gauge-mediated scenario, the VEV $\mathcal{F}_Z$ of the $Z$ field can be the primary source of supersymmetry breaking. The supersymmetry-breaking effect in the $Z$ field can be transmitted to all the superparticles through gauge interactions [47–49], and no additional singlet is necessary. In this case, both $A$ and $B$ parameters are of the order of the gravitino mass (i.e., $\Lambda^2 \simeq m_{3/2} M_P$). Since in the gauge-mediated scenario $m_{3/2}$ is, in general, much smaller than that in the supergravity-mediated one, the $A$ and $B$ parameters are certainly below the electroweak scale. Thus, the neutrino Yukawa coupling in Eq. (51) cannot give neutrino masses in the range of the various $\Delta m^2$ suggested by different experiments, except

[7] If the discrete symmetry in Sec. IV is generic, i.e., $Z_n$ with $n \neq 2$, it is possible to have $M_R \simeq 0$ and $M_R B \lesssim m_{\text{weak}}^2$. Then, the neutrino mass matrix takes the form $\sim ((m_L, m_D)^T, (m_D, 0)^T)$, where $m_L$ is generated by the diagram shown in Fig. 2, and it is suppressed with respect to $m_{\text{weak}}$ only by a numerical loop factor if $A^2 \simeq (M_R B) \simeq m_{\text{weak}}^2$.

perhaps for the vacuum-oscillation region of the solar neutrino experiments. A substantial contribution to neutrino masses may come from the radiative mechanism. Notice that all the expressions for $m_L$ and $m_D$ derived in Sec. V still apply to the gauge-mediated case. However, since the values of $A$ and $B$ are now smaller, the values of $M_R$ and of the $(B-L)$-breaking scale $M_G$ needed to obtain sizable neutrino masses are also lower than those needed in the supergravity-mediated case.

It should be noticed here that, if the $Z_n$ charge $-1$ is assigned not only to the $\bar{N}$ field but also to all SU(2)-singlet SM fields $\bar{U}, \bar{D}$, and $\bar{E}$, then all the SM Yukawa couplings are forbidden and effective Yukawa operators are generated also for quark and charged-lepton fields through higher-dimensional operators such as those in Eq. (2). The tree-level contributions that these operators give to Yukawa couplings of quarks and charged leptons are, however, completely negligible. Thus, their masses, as well as Yukawa couplings, are generated radiatively, as in the framework of Ref. [33], through trilinear supersymmetry-breaking parameters induced by $\mathcal{F}_Z$, and gluino and/or neutralino masses. In this case, a unified radiative generation picture for quark and lepton masses emerges. The smallness of neutrino masses with respect to the masses of all other fermions is then easily explained by the fact that neutrinos feel the presence of neutralinos and sneutrinos at the large scales $M_G$ and $M_R$ of $U(1)_{B–L}$ violation. In contrast, the leading contribution to quark and charged-lepton masses comes from the exchange of sfermions and gluinos or sfermions and neutralinos at the electroweak scale.

## IX. CONCLUSIONS

In this paper, two different mechanisms to obtain light physical neutrino states in supersymmetric models with three right-handed neutrinos are proposed. It is observed that if the lowest order neutrino Yukawa operators are forbidden by means of a horizontal discrete symmetry, Yukawa couplings are still induced by higher-dimensional operators suppressed by the Planck mass. In these operators, the three fields that participate in a Yukawa interaction are coupled to a spurion field $Z$, which acquire a supersymmetry-conserving and a supersymmetry-violating VEV, respectively, $\mathcal{A}_Z$ and $\mathcal{F}_Z$. If a large hierarchy between the two VEV's is possible, then naturally small Yukawa couplings can be induced at the tree level.

Models where such a hierarchy can be implemented exist and one is explicitly described in this paper. In this model, the $Z$ direction is flat at the tree level. This flatness is lifted by loop corrections, and a supersymmetry-conserving VEV $\mathcal{A}_Z \sim 16\pi^2 m_{3/2}$ is induced by supersymmetry breaking. The strong suppression of neutrino Yukawa couplings $y_\nu$ gets naturally linked to the hierarchy between $m_{3/2}$ and $M_P$: $y_\nu \sim \mathcal{A}_Z/M_P \sim 16\pi^2(m_{3/2}/M_P)$. If Majorana masses for active and sterile neutrinos are forbidden by an additional symmetry such as, for example, lepton number, only Dirac neutrino masses are generated. They are of order $\sim \mathcal{A}_Z v/M_P$, with $v$ an electroweak VEV, i.e., they can reach $\sim 10^{-2}\,\mathrm{eV}$ if all interaction couplings are assumed of $\mathcal{O}(1)$. Intergenerational mass splittings and related oscillations rely on specific textures of these couplings.

Nevertheless, it is possible to forbid only the lowest order mass term for right-handed neutrinos, but leave allowed higher-dimensional mass operators for left- as well as right-handed neutrinos (see the second tree-level scenario discussed in Sec. II). Besides the Dirac mass $m_D$ given above, also Majorana masses for active and sterile neutrino $m_L$ and $M_R$ are present, respectively of order $\sim v^2/M_P$ and $\sim \mathcal{A}_Z^2/M_P$. Six physical states of Majorana type are generated: $\nu_i$ and $n_i$ $(i = 1, 2, 3)$. The states $n_i$ correspond mainly to sterile neutrinos and are the heaviest ones. For all interaction couplings of $\mathcal{O}(1)$, their mass is $10^4$–$10^6$ times larger than the mass of the states $\nu_i$, which is of order $\sim 10^{-5}\,\mathrm{eV}$. Different patterns for different generations may be induced by allowing a nontrivial structure of the interaction couplings.

The second mechanism proposed is that of the radiative generation of neutrino masses via sneutrino-neutralino loops. It is illustrated in the supersymmetrized version of a typical model that leads to the traditional seesaw mechanism: i.e., a supersymmetric model with three right-handed neutrinos and an additional gauge interaction broken at a large scale $M_G$. Nevertheless, large radiative contributions to the Majorana mass $m_L$ may arise also in models without any additional gauge interaction, but with an explicit violation of lepton number. Also for this mechanism, as in the tree-level one, the chief assumption is the vanishing of the lowest-order neutrino Yukawa operators.

It is shown that (i) the Majorana neutrino mass $M_R$ for right-handed neutrinos is not necessarily of order $\mathcal{O}(M_G)$ and can even be exactly vanishing, that (ii) a seesaw pattern of light-heavy scales is present in the neutralino and (possibly) the sneutrino mass matrix. Dirac and Majorana masses $m_D$ and $m_L$ are induced by trilinear and bilinear soft sneutrino terms as well as neutralino mass terms, which provide, respectively, information on chirality breaking, on lepton-number violation and on the correct number of $R$ charges. Apart from a loop suppression factor, ratios of light over heavy scales, present in the neutralino and sneutrino mass matrices, steer $m_D$ and $m_L$ away from the electroweak scale. Since these ratios enter in the expression for $m_D$ and $m_L$ with powers larger than 1, their effectiveness in suppressing $m_D$ and $m_L$ is stronger than in the usual seesaw mechanism, and the scale $M_G$ can be much lower.

Such a mechanism allows a large variety of possible neutrino spectra. In the case of exact vanishing of $M_R$, three Dirac neutrino states are possible, as in the first scenario of the tree-level mechanism or in a traditional Higgs mechanism with highly tuned Yukawa couplings. These states can have now arbitrary mass depending on the value of $M_G$. In this case, a texture of neutrino masses consistent with experimental observations finds its origin in the texture of the trilinear $A$ terms for sneutrinos.

In general, for nonvanishing $M_R$, six Majorana states $\nu_i$ and $n_i$, are generated. For heavy $M_R$, the three states $n_i$ are heavy and no space is left for light sterile neutrinos. For light $M_R$, the situation is very rich. The six states are all light. The three $n_i$ neutrinos, however, can still be much heavier than the $\nu_i$'s, in which case the mixing angles between $n_i$'s and $\nu_i$'s are very small. There are finally the other interesting cases in which the $\nu_i$ and $n_i$ states are roughly at the same scale or nearly degenerate, with mixing angles ranging from small to maximal. Flavor oscillations can be accommodated in this class of scenarios, as before, i.e., relying on specific textures of the trilinear and bilinear neutrino soft parameters. Oscillations among active and sterile neutrinos are, however, also possible. Whether this is the answer to the puzzle posed by the present results of solar, atmospheric, and reactor neutrino experiments is a question which new experimental data will answer, hopefully, soon. Irrespective of this, it is nontrivial that such oscillations, as well as those between active and much heavier sterile neutrinos advocated in supernova physics, can be easily accommodated in this class of scenarios.

Since neutrino masses are much smaller than all other masses, it is plausible to assume that the lowest-order neutrino Yukawa operators inducing neutrino masses are vanishing and that the origin of neutrino masses is linked to operators of higher dimensionality or to radiative generation. We have shown that both possibilities are easily implemented in supersymmetric models. The resulting mechanisms encompass the typical Higgs and seesaw mechanisms, although with different realizations, and give also rise to various interesting neutrino spectra.


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**Note added** While this work was being completed, we received Ref. [50], which makes another proposal for small neutrino masses in supersymmetric models.

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